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A NOTE ON LINEAR PROGRAMMING AND THE SINGLE MACHINE  
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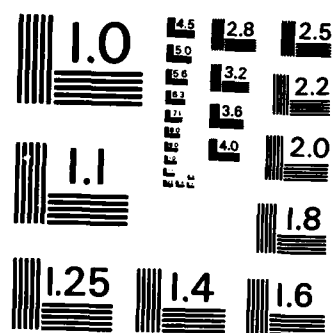
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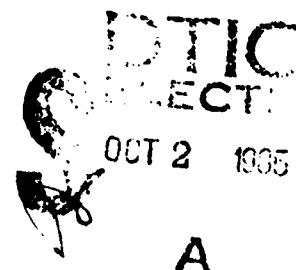
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DEPARTMENT OF  
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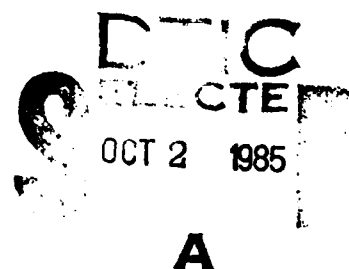
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## Abstract

### A Note on Programming and the Single Machine Lotsize Scheduling Problem

by

Thom J. Hodgson

Henry L. W. Nuttle

✓ Past research on the lotsize scheduling problem has relied heavily on the following assumptions:

1. Each product is to be produced on a regular invariant cycle.
2. The inventory level at the onset of each production run is zero.

In this paper these two restrictions have been dropped. It is shown that the inventory cost function can be expressed as a linear function. This allows the use of linear programming methods to determine the optimal lengths for a given sequence of production runs.

*Additional keywords:*  
*Industrial Engineering Inventory Analysis*

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The lotsize scheduling problem is basically that of finding a minimum cost production schedule on a single machine, for  $N$  different products, each with known demand rate, production rate, and setup time. As its name implies, the problem has elements of both inventory and scheduling. The objective function, the sum of setup and inventory costs, is derived from inventory theory; and the feasibility requirement associated with machine availability is primarily a scheduling problem.

Variations of the problem have been the subject of numerous articles in the literature. A comprehensive bibliography through 1976 is given in [4]. More recent references include [1]-[3],[5]-[7], [9], and [11].

The most frequently considered variation is the one in which the objective is to determine a cyclic, repeatable schedule which minimizes setup plus inventory costs per unit time. In this context one is faced with determining the sequence in which products are to be processed and the length of each production run.

In this paper we focus on determining the run lengths, given the sequence. In particular, we show that when all runs for a given product must be of equal length, but production may start before inventory is exhausted, then the optimal cycle and run lengths may be determined by linear programming methods. This result can be used in conjunction with a sequence generating procedure to provide solutions to the overall problem. It can also be used to easily "tune" schedules produced by other procedures to be more acceptable from the standpoint of implementation.

Our model corresponds to Maxwell's [8] when the "equal lots rule" is used and idle time between the production runs is permitted. Relative to the model of Delporata and Thomas [3], it requires equal lots for each run of a given product, but allows non-zero inventories at the beginning of the runs.

Consider a set of  $N$  products to be produced in lots according to a repeating cyclic schedule on a single machine which can process one product at a time. Let  $d_k$ ,  $p_k$ , and  $S_k$  denote demand rate, production rate, and set up time, respectively, for product  $k$ .

Suppose that we are given a sequence  $J = \{k_1, \dots, k_j, \dots, k_L\}$  of production runs to be performed in a cycle of length  $T$  (to be determined) time units.

$k_j \in \{1, 2, \dots, N\}$  denotes the product to be processed during run  $j$  and  $n_k$  denotes the cardinality of the set  $\{j \in \{1, 2, \dots, L\} | k_j = k\}$ , i.e., the number of runs of product  $k$ . Thus  $L = \sum_{k=1}^N n_k$ . We require all production runs for product  $k$  to yield  $d_k T / n_k$  units which implies a run length of  $d_k T / p_k n_k$  time units.

In this context, we wish to determine, the cycle length,  $T$ , and the production run start times (after setup),  $X_j$ ,  $0 \leq X_j \leq T$ ,  $j = 1, \dots, L$ , so as to minimize the cost (setup plus inventory carrying) per unit time, subject to the restriction that all demand be satisfied without backordering.

If  $W_j$  denotes the amount of idle time following production run  $j$ , then start times, idle times, and cycle length are related by

$$X_j + \frac{d_k T}{p_k n_{k_j}} + W_j + S_{k_{j+1}} = X_{j+1}, \quad j = 1, \dots, L-1 \quad (1)$$

$$X_L + \frac{d_k T}{p_k n_{k_L}} + W_L = T \quad (2)$$

where  $X_1 = S_{k_1}$ .

Let  $z_j$  be the index of the next production run of product  $k_j$  after production run  $j$ . If  $I_j$  is the inventory level of product  $k_j$  at the beginning of run  $j$ , then

$$I_j + d_k T / n_{k_j} - d_k (X_{z_j} - X_j) = I_{z_j}, \quad j = 1, 2, \dots \quad (3)$$

along with  $I_j \geq 0$ , for all  $j$ , assures that no shortages will occur. Note that equation (3) is not needed when production run  $j$  is the  $n_k^j$ th production run for product  $k_j$ , since only  $n_k^j - 1$  equations are necessary to fully define the relationships between the  $n_k^j$  inventory levels.

Our objective is to minimize the total setup plus carrying cost per unit time. If inventory levels,  $I_j$ , were required to be zero, then this cost would be given by

$$\sum_{k=1}^N (n_k A_k / T + h c_k (1 - d_k / p_k) d_k T / 2 n_k) \quad (4)$$

where  $A_k$  and  $c_k$  are the setup and piece costs, respectively, for product  $k$ , and  $h$  is the inventory carrying charge. The associated inventory pattern for an arbitrary product  $k$  ( $n_k = 3$ ) is shown in Figure 1a. When the  $I_j$ 's are allowed to be positive (implying that production of a product may begin before the current supply is exhausted) inventory patterns such as that illustrated in Figure 1b (where we use the notation  $I_{j_i}$  to represent the level for product  $k$  at the beginning of its  $i$ th run) are then possible. Such patterns result in an increased carrying cost (versus  $I_{j_i} = 0$ ) which is proportional to the area (A+B) in Figure 1b (or its equivalent for other patterns). Since the production schedule will be repeated every  $T$  time units, the increase in carrying cost for product  $k$  per unit time may be calculated as  $h c_k (A+E)/T$  (or its equivalent for other patterns). As illustrated in Figure 1c, this increase is easily shown to be

$$h c_k \sum_{i=1}^{n_k} I_{j_i} / n_k \quad (5)$$

Note that the increase is linearly dependent on the minimum inventory levels.



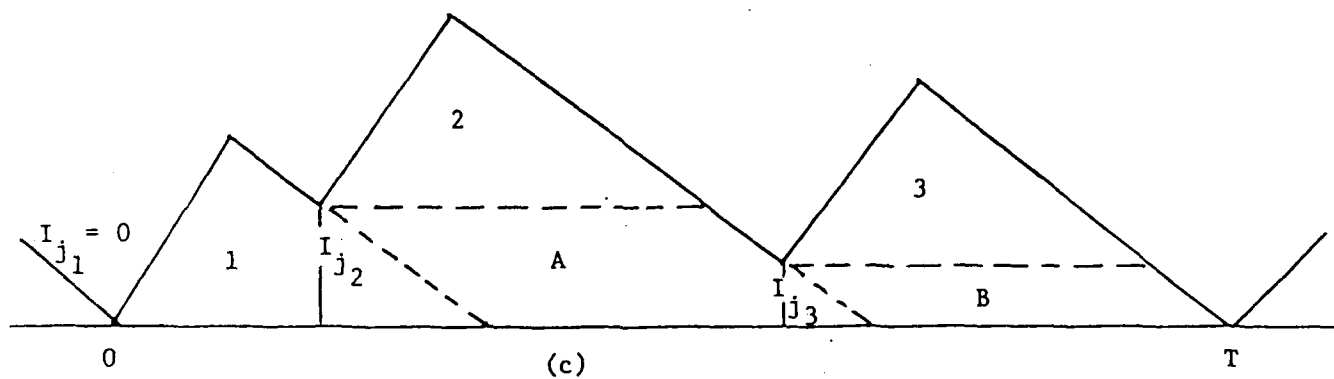
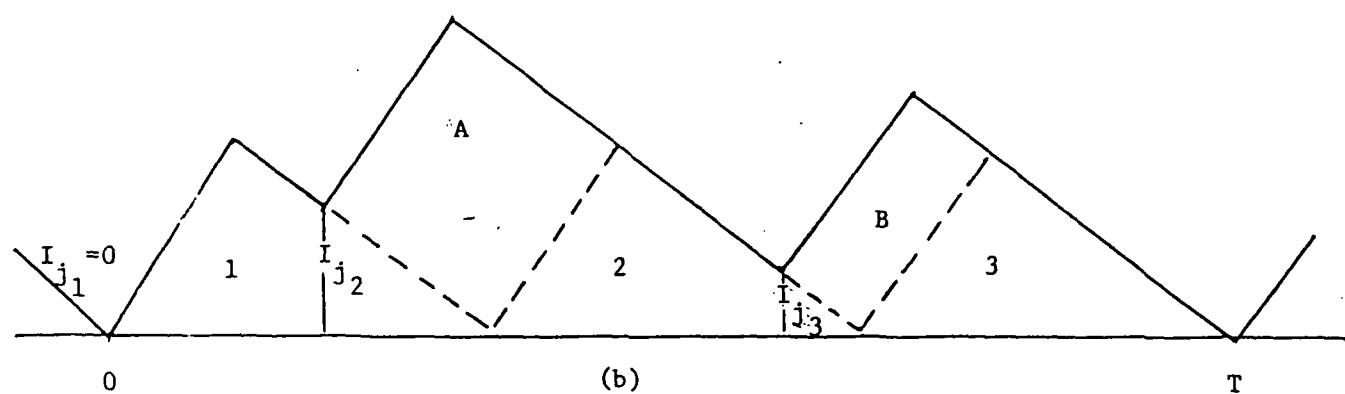
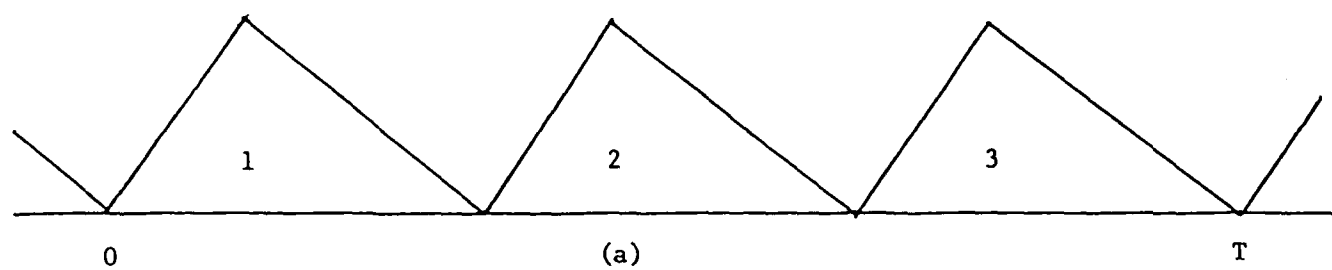


FIGURE 1  
Modified Inventory Levels

Based on equation (5), the cost expression ((4)) may be rewritten as

$$\sum_{k=1}^N n_k A_k / T + h \left[ \sum_{k=1}^N c_k (1 - d_k / p_k) d_k T / 2 n_k + \sum_{j=1}^L c_k I_j / n_{k_j} \right] \quad (6)$$

Minimizing (6) subject to (1) - (3) plus nonnegativity restrictions on the  $X_j$ 's,  $I_j$ 's,  $W_j$ 's and  $T$  will yield an optimal schedule.

This problem may be solved by linear programming methods. Note that the problem may be restated as, choose  $T \geq 0$  to

$$\min Z(T) = A(T) + F(T) \quad (7)$$

where  $A(T) = \sum_{k=1}^N n_k A_k / T$  and

$$F(T) = \min h \left[ \sum_{k=1}^N c_k (1 - d_k / p_k) d_k T / 2 n_k + \sum_{j=1}^L c_k I_j / n_{k_j} \right]$$

subject to (1)-(3) plus nonnegativity, which is a linear program.

For  $T \geq 0$ , the hyperbolic setup cost function,  $A(T)$ , is convex, while  $F(T)$  is piecewise linear and convex [10]. Thus  $Z(T)$  is convex for  $T \geq 0$ .

Let  $T^0$  be the point where  $F(T)$  attains its minimum. For  $T < T^0$  both  $A(T)$  and  $F(T)$ , and thus  $Z(T)$ , are decreasing. Thus the optimum  $T$  will be at least as large as  $T^0$ , so that we can restrict our attention to  $T \geq T^0$ . For this range  $F(T)$  can be obtained by adding the constraint

$$T \geq T^0 \quad (8)$$

to the linear program and applying sensitivity analysis to  $T^0$ . For  $T \geq T^0$ ,  $F(T)$  is piecewise linear and non decreasing with break points corresponding to optimal basis changes in the LP. Thus, the minimum of (7) will occur either at one of the break-

points of  $F(T)$  or in between breakpoints. To determine if the minimum occurs between two breakpoints, the following procedure can be used. The expression for the variable elements of the total cost function in a given segment is just

$$\sum_{k=1}^N n_k A_k / T + ST, \quad (9)$$

where  $S$  is the sensitivity to  $T$  (dual variable for (8)) of the linear program objective function in the segment. The minimum of (9) occurs at

$$T' = \left[ \frac{\sum_{k=1}^N n_k A_k}{S} \right]^{1/2} \quad (10)$$

If  $T'$  falls within the boundaries of the segment, then the minimum of  $Z(T)$  occurs at  $T'$ . Since  $S$  is non decreasing with  $T^0$ , if  $T'$  falls below the lower end of the segment, then the optimal  $T$  occurs at the lower end of the segment. If  $T'$  falls above the upper end of the segment, the next segment must be investigated.

If we ignore the requirement that only one product be processed at a time, the least cost value of  $T$  can be determined as

$$T^* = \left[ \frac{2 \sum_{k=1}^N n_k A_k}{h \sum_{k=1}^N c_k d_k (1 - d_k / p_k) / n_k} \right]^{1/2} \quad (11)$$

When this requirement is considered, it is easy to see that the optimal value of  $T$  will be no less than  $T^*$ . Thus we may replace constraint (8) with

$$T \geq \max (T^0, T^*)$$

Finally, we should note that, from the standpoint of implementation, the production quantities associated with the optimal value of  $T$  may be impractical. For example, if  $T$  turns out to be 19.335 workdays, this may call for producing 2 lots of a product each of which covers the demand for 9.6675 workdays. As an alternative, albeit at some cost relative to optimality, we may simply specify a  $T''$  (for example, 20 workdays) and solve the LP with  $T = T''$ . In this context, the optimal  $T$  can be used to determine the "ballpark" for the choice of  $T''$ .

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